MATH162 – Mathematics 1E, Part 2

Summer Session 2006/2007

Mid-Session Test

*Student Name:*____________________________ *Student Number:*____________

*Tutorial Group:*____ *Tutorial Time:*____________ *Tutor:*____________

Instructions

Time Allowed: 90 minutes

Do NOT remove the Answer Sheet from the Question Section.

- *All questions are to be attempted:*
  25 Multiple choice questions (25 marks).
- *Working is to be done in the Question Section.*
- *Answers are to be marked on the page entitled Answer Sheet by completely shading the box with the letter of your chosen response.*
- *Answers should also be marked on the Question Section.*
- *Write your name on both the Question Section and the Answer Sheet.*
- *At the end of the test return the complete test paper, including the Answer Sheet.*

Calculators are permitted.

A one-page, A4-sized, double-sided summary sheet is permitted.

A Table of Integrals is attached.

This test paper is not to leave this room.

1. When the Midpoint Rule is applied to get an approximate value for the integral

$$\int_0^2 (2x^2 - x)dx$$

the absolute error which results is

(a) $\frac{1}{6}$.

(b) $\frac{5}{6}$.

(c) $\frac{7}{3}$.

(d) $\frac{10}{3}$.

(e) none of these.

2. When the Two-Point Gauss Rule is used to find an approximation to

$$\int_0^1 e^{x^2}\, dx$$

the value obtained, to four decimal places, is

(a) 1.2840.

(b) 1.4542.

(c) 0.7466.

(d) 0.7788.

(e) none of these.

3. Consider the following quadrature scheme

$$\int_0^1 f(x)dx \approx \omega_1 f\left(\frac{1}{4}\right) + \omega_2 f\left(\frac{3}{4}\right).$$

If the weights ω_1 and ω_2 are chosen using the standard method of integrating polynomials exactly, then

(a) $\omega_1 = -2$ and $\omega_2 = 2$.

(b) $\omega_1 = \omega_2 = \frac{1}{2}$.

(c) $\omega_1 = \frac{1}{3}$ and $\omega_2 = \frac{5}{9}$.

(d) $\omega_1 = \frac{3}{4}$ and $\omega_2 = \frac{1}{4}$.

(e) none of these.

4. If a compound Trapezoidal rule with 4 subintervals is applied to estimate

$$\int_0^{\pi} \sin x \, dx,$$

then the value obtained is

(a) $\frac{\pi^2}{4}$.

(b) $\frac{1}{2}\left(1+\sqrt{2}\right)$.

(c) $\frac{\pi}{4}\left(1+\sqrt{2}\right)$.

(d) $\frac{\pi}{8}\left(1+\sqrt{2}\right)$.

(e) $\frac{\pi}{2}$.

5. When the Simpson's rule is applied to get an approximate value for the integral

$$\int_0^1 x^6 dx$$

the value obtained is

(a) $\frac{85}{486}$.

(b) $\frac{41}{271}$.

(c) $\frac{1}{6}$.

(d) $\frac{4}{21}$.

(e) $\frac{17}{96}$.

6. The value of $\int_{-\pi}^{\pi} \cos mx \cos nx \; dx$ is equal to

(a) $\begin{cases} 0, & m \neq n \\ \pi, & m = n. \end{cases}$

(b) $\begin{cases} \pi, & m \neq n \\ 0, & m = n. \end{cases}$

(c) 0.

(d) π.

(e) none of these.

7. The value of $\int_0^{\frac{\pi}{4}} x^2 \sin 2x \; dx$, to 3 decimal places, is equal to

(a) 0.

(b) 0.25.

(c) 0.393.

(d) 0.143.

(e) none of these.

8. The value of $\int_0^1 \frac{x}{\sqrt{1+x^2}}dx$ is

(a) $\frac{1}{\sqrt{2}}$.

(b) 1.

(c) 1.4142.

(d) $\sqrt{2}-1$.

(e) non-existent.

9. The value of $\int_0^{\frac{\pi}{2}} \frac{3\ dx}{2\cos x + 3\sin x + 2}$, to 3 decimal places, is equal to

(a) 3.296.

(b) 2.749.

(c) 0.916.

(d) 1.

(e) none of these.

10. The integral $\int \frac{dx}{x^2+5x-6}$ is equal to

(a) $\frac{1}{7(x-1)} - \frac{1}{7(x+6)} + c$.

(b) $\frac{1}{5}\ln|x+3| - \frac{1}{5}\ln|x-2| + c$.

(c) $\frac{1}{7}\ln|x-1| - \frac{1}{7}\ln|x+6| + c$.

(d) $\frac{1}{5}\ln|x-2| - \frac{1}{5}\ln|x+3| + c$.

(e) $\frac{1}{7}\ln|x+6| - \frac{1}{7}\ln|x-1| + c$.

11. The integral $\int \tan x \sec^2 x\ dx$ is equal to

(a) $\frac{1}{2}\sec^2 x + c$.

(b) $\frac{1}{2}\tan^2 x \sec x + c$.

(c) $\frac{1}{2}\tan^2 x \sec^2 x + c$.

(d) $\frac{1}{6}\tan^2 x \sec^3 x + c$.

(e) none of these.

12. The integral $\int \sqrt{4-x^2}\ dx$ is equal to

(a) $2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + c.$

(b) $\frac{1}{2}x\sqrt{4-x^2} - 2\ln\left|x + \sqrt{4-x^2}\right| + c.$

(c) $\frac{2(4-x^2)^{3/2}}{3} + c.$

(d) $2x - \frac{x^2}{2} + c.$

(e) none of these.

13. The integral $\int \cos(\ln x)\ dx$ is equal to

(a) $\frac{x}{2}\left[\cos(\ln x) + \sin(\ln x)\right].$

(b) $\ln x \sin x + \frac{1}{x}\cos x + c.$

(c) $\ln x \cos x + \frac{1}{x}\sin x + c.$

(d) $\sin(\ln x) + c.$

(e) $\sin\left(\frac{1}{x}\right) + c.$

14. The integral $\int x\cos(x^2)\ dx$ is equal to

(a) $\frac{x^2}{2}\cos(x^2) + x\sin(x^2) + c.$

(b) $x\sin(x^2) + c.$

(c) $\frac{1}{2}\sin^2 x + c.$

(d) $\frac{1}{2}\sin(x^2) + c.$

(e) none of these.

15. The integral $\int \sin^2 x \cos 3x\ dx$ is equal to

(a) $\frac{2\sin x\cos 3x}{\cos x} + \frac{\sin^2 x\sin 3x}{3} + c.$

(b) $\frac{1}{2}\left[\frac{\sin 3x}{3} + \frac{\sin x}{2} + \frac{\sin 5x}{10}\right] + c.$

(c) $\frac{1}{2}\left[\frac{\sin 3x}{3} - \frac{\sin x}{2} - \frac{\sin 5x}{10}\right] + c.$

(d) $\cos^2 x\sin 3x + c.$

(e) none of these.

16. Consider the integral

$$\int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}}.$$

After rationalising the denominator of the integrand, we find the integral is equal to

(a) $\dfrac{\sqrt{2x}}{3}+c.$

(b) $\dfrac{(x+1)^{3/2}}{3}+\dfrac{(x-1)^{3/2}}{3}+c.$

(c) $\dfrac{1}{2}\left[\sqrt{x+1}+\sqrt{x-1}\right]+c.$

(d) $\dfrac{1}{2}\left[\sqrt{x+1}+\sqrt{x-1}\right]+c.$

(e) $\dfrac{1}{2}\left[\left(\sqrt{x+1}\right)^3+\left(\sqrt{x-1}\right)^3\right]+c.$

17. To evaluate the integral $\displaystyle\int \frac{dx}{x^2+2x+2}$, the most appropriate method is to

(a) let $u = x^2+2x+2$.

(b) use a standard integral.

(c) use the method of integration by parts.

(d) factorise the denominator and then find the partial fraction decomposition.

(e) complete the square of the denominator and then make a substitution.

18. The form of the partial fraction decomposition of $\dfrac{5x^2-x+1}{(x-1)(x^2-x-3)^2}$ is given by

(a) $\dfrac{A_1}{x-1}+\dfrac{A_2x+B_1}{(x^2-x-3)^2}.$

(b) $\dfrac{A_1}{x-1}+\dfrac{A_2x+B_1}{x^2-x-3}+\dfrac{A_3x+B_2}{(x^2-x-3)^2}.$

(c) $\dfrac{A_1}{x-1}+\dfrac{A_2x+B_1}{(x^2-x-3)^2}+\dfrac{A_3x+B_2}{(x^2-x-3)^2}.$

(d) $\dfrac{A_1}{x-1}+\dfrac{A_2}{x^2-x-3}+\dfrac{A_3}{(x^2-x-3)^2}.$

(e) $\dfrac{A_1}{x-1}+\dfrac{A_2}{(x^2-x-3)^2}+\dfrac{A_3}{(x^2-x-3)^2}.$

19. The integral $\displaystyle\int \frac{dx}{\sqrt{4x^2+24x+61}}$ can be transformed to

(a) $\displaystyle\int \frac{du}{\sqrt{u}}$ where $u = 4x^2+24x+61$.

(b) $\displaystyle\frac{1}{2}\int \frac{du}{u+\frac{5}{2}}$ where $u = x+3$.

(c) $\displaystyle\frac{1}{2}\int \frac{du}{u^2+\frac{25}{4}}$ where $u = x+3$.

(d) $\displaystyle\frac{1}{4}\int \frac{du}{\sqrt{u^2+\frac{25}{4}}}$ where $u = x+3$.

(e) $\displaystyle\frac{1}{2}\int \frac{du}{\sqrt{u^2+\frac{25}{4}}}$ where $u = x+3$.

20. If $z = -1 + i\sqrt{3}$, then z^{12} is equal to

(a) -24.

(b) 24.

(c) -2^{12}.

(d) 2^{12}.

(e) $2^{11}\left(-1 + i\sqrt{3}\right)$.

21. Given that $z_1 = 1 - i$ and $z_2 = -2 + 4i$, the values of $\alpha = 2z_2 - 3\overline{z_1}$ and $\beta = \mathrm{Im}\left(\frac{z_1}{z_2}\right)$ are

(a) $\alpha = -7 + 5i,\ \beta = \frac{-i}{10}$.

(b) $\alpha = -7 + 5i,\ \beta = \frac{-1}{10}$.

(c) $\alpha = -7 + 11i,\ \beta = \frac{-i}{10}$.

(d) $\alpha = -7 + 11i,\ \beta = \frac{-1}{10}$.

(e) none of these.

22. The six sixth roots of unity are given by

(a) $1, -i, \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right), \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right), \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right), \left(-\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$.

(b) $1, -i, \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right), \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right), \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right), \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$.

(c) $1, -1, \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right), \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right), \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right), \left(-\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$.

(d) $1, -1, \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right), \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right), \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right), \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$.

(e) none of these.

23. The value of $\lim\limits_{x \to -5^-} \frac{\sqrt{(5+x)^2}}{x+5}$ is

(a) non-existent.

(b) 10.

(c) 1.

(d) 0.

(e) -1.

24. The value of $\lim_{x \to 0} \dfrac{2\sin x - \sin 2x}{4x^3}$ is

(a) non-existent.

(b) 1.

(c) $\dfrac{1}{2}$.

(d) $\dfrac{1}{4}$.

(e) 0.

25. The function f is defined by

$$f(x) = \begin{cases} ax + a, & x < 1 \\ 3, & 1 \le x < 2 \\ cx^2, & 2 \le x. \end{cases}$$

The function is continuous only when we choose

(a) $a = \frac{3}{2}$; $c = \frac{3}{4}$.

(b) $a = \frac{2}{3}$; $c = \frac{4}{3}$.

(c) $a = 3 = c$.

(d) values of a and c different from these.

(e) no values for a and c, as none make f continuous.

TABLE OF INTEGRALS

[1] $$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

[2] $$\int \frac{dx}{x} = \ln|x| + c$$

[3] $$\int e^x\, dx = e^x + c$$

[4] $$\int \sin x\, dx = -\cos x + c$$

[5] $$\int \cos x\, dx = \sin x + c$$

[6] $$\int \tan x\, dx = \ln|\sec x| + c$$

[7] $$\int \sec^2 x\, dx = \tan x + c$$

[8] $$\int \operatorname{cosec}^2 x\, dx = -\cot x + c$$

[9] $$\int \sinh x\, dx = \cosh x + c$$

[10] $$\int \cosh x\, dx = \sinh x + c$$

[11] $$\int \tanh x\, dx = \ln(\cosh x) + c$$

[12] $$\int x(ax+b)^n\, dx = \frac{1}{a^2}(ax+b)^{n+1}\left[\frac{ax+b}{n+2} - \frac{b}{n+1}\right] + c, \quad n \neq -1, -2$$

[13] $$\int \frac{x^2}{ax+b}\, dx = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2 \ln|ax+b|\right] + c$$

[14] $$\int \frac{x^2}{(ax+b)^2}\, dx = \frac{1}{a^3}\left[ax+b - \frac{b^2}{ax+b} - 2b\ln|ax+b|\right] + c$$

[15] $$\int x\sqrt{ax+b}\, dx = \frac{2}{a^2}\left[\frac{(ax+b)^{5/2}}{5} - \frac{b(ax+b)^{3/2}}{3}\right] + c$$

[16] $$\int \frac{x}{\sqrt{ax+b}}\, dx = \frac{2ax-4b}{3a^2}\sqrt{ax+b} + c$$

[17] $$\int \frac{1}{x\sqrt{ax+b}}\, dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + c, \quad b > 0$$

[18] $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

[19] $$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

[20] $$\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + c$$

[21] $$\int \frac{1}{(a^2-x^2)^2}\,dx \;=\; \frac{x}{2a^2(a^2-x^2)}+\frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right|+c$$

[22] $$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx \;=\; -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|+c$$

[23] $$\int \frac{1}{(a^2-x^2)^{3/2}}\,dx \;=\; \frac{1}{a^2}\frac{x}{\sqrt{a^2-x^2}}+c$$

[24] $$\int \frac{\sqrt{a^2-x^2}}{x}\,dx \;=\; \sqrt{a^2-x^2}-a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|+c$$

[25] $$\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx \;=\; \ln\left|x+\sqrt{x^2\pm a^2}\right|+c$$

[26] $$\int \frac{1}{x\sqrt{x^2+a^2}}\,dx \;=\; -\frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|+c$$

[27] $$\int \frac{1}{(x^2\pm a^2)^{3/2}}\,dx \;=\; \pm\frac{1}{a^2}\frac{x}{\sqrt{x^2\pm a^2}}+c$$

[28] $$\int \sqrt{x^2\pm a^2}\,dx \;=\; \frac{1}{2}x\sqrt{x^2\pm a^2}\pm\frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|+c$$

[29] $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx \;=\; \sqrt{x^2+a^2}-a\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|+c$$

[30] $$\int \frac{1}{b+ke^{ax}}\,dx \;=\; \frac{1}{ab}\left[ax-\ln(b+ke^{ax})\right]+c, \quad ab\neq 0$$

[31] $$\int e^{ax}\sin bx\,dx \;=\; \frac{1}{a^2+b^2}e^{ax}(a\sin bx-b\cos bx)+c$$

[32] $$\int e^{ax}\cos bx\,dx \;=\; \frac{1}{a^2+b^2}e^{ax}(a\cos bx+b\sin bx)+c$$

[33] $$\int \sin^n x\,dx \;=\; -\frac{1}{n}\cos x\sin^{n-1}x+\frac{n-1}{n}\int\sin^{n-2}x\,dx$$

[34] $$\int \cos^n x\,dx \;=\; \frac{1}{n}\sin x\cos^{n-1}x+\frac{n-1}{n}\int\cos^{n-2}x\,dx$$

[35] $$\int \tan^n x\,dx \;=\; \frac{1}{n-1}\tan^{n-1}x-\int\tan^{n-2}x\,dx$$

[36] $$\int \sec^n x\,dx \;=\; \frac{\sec^{n-2}x\tan x}{n-1}+\frac{n-2}{n-1}\int\sec^{n-2}x\,dx$$

[37] $$\int \sin^m x\cos^n x\,dx \;=\; \frac{\sin^{m+1}x\cos^{n-1}x}{m+n}+\frac{n-1}{m+n}\int\sin^m x\cos^{n-2}x\,dx$$

[38] $$\int x^n e^x\,dx \;=\; x^n e^x-n\int x^{n-1}e^x\,dx$$

[39] $$\int x^n\sin x\,dx \;=\; -x^n\cos x+n\int x^{n-1}\cos x\,dx$$

[40] $$\int x^n\cos x\,dx \;=\; x^n\sin x-n\int x^{n-1}\sin x\,dx$$

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Answer Sheet 3

Completely fill in the appropriate box for each question: ■

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E

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Answer Sheet 3

Completely fill in the appropriate box for each question: ■

1.	■	■	C	■	■
2.	■	B	■	■	■
3.	■	B	■	■	■
4.	■	■	C	■	■
5.	■	■	■	■	E
6.	A	■	■	■	■
7.	■	■	■	D	■
8.	■	■	■	D	■
9.	■	■	C	■	■
10.	■	■	C	■	■
11.	A	■	■	■	■
12.	A	■	■	■	■
13.	A	■	■	■	■
14.	■	■	■	D	■
15.	■	■	C	■	■
16.	■	B	■	■	■
17.	■	■	■	■	E
18.	■	B	■	■	■
19.	■	■	■	■	E
20.	■	■	■	D	■
21.	■	B	■	■	■
22.	■	■	■	D	■
23.	■	■	■	■	E
24.	■	■	■	D	■
25.	A	■	■	■	■