

**MATH141 - Worksheet
SURDS**

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y} \quad \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} \quad a\sqrt{x} \pm b\sqrt{x} = (a \pm b)\sqrt{x}$$

$$\sqrt{x} \times \sqrt{x} = x \quad a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c \text{ and } b = d$$

$(\sqrt{x} + \sqrt{y})$ and $(\sqrt{x} - \sqrt{y})$ are **conjugate surds**
 $(\sqrt{x} + \sqrt{y}) \times (\sqrt{x} - \sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = x - y$
Rationalising the denominator:
 $\frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$
 $\frac{1}{\sqrt{x} + \sqrt{y}} = \frac{1}{\sqrt{x} + \sqrt{y}} \times \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \frac{\sqrt{x} - \sqrt{y}}{(\sqrt{x})^2 - (\sqrt{y})^2} = \frac{\sqrt{x} - \sqrt{y}}{x - y}$

1. Express each of the following in simplest surd form.

For example $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

- | | | | |
|-------------------|-----------------------------|-----------------------------|-----------------------------|
| (a) $\sqrt{8}$ | (b) $\sqrt{20}$ | (c) $\sqrt{24}$ | (d) $\sqrt{27}$ |
| (e) $\sqrt{72}$ | (f) $\sqrt{75}$ | (g) $\sqrt{84}$ | (h) $\sqrt{63}$ |
| (i) $\sqrt{180}$ | (j) $\sqrt{200}$ | (k) $\sqrt{288}$ | (l) $3\sqrt{54}$ |
| (m) $5\sqrt{125}$ | (n) $\frac{1}{2}\sqrt{320}$ | (o) $\frac{1}{5}\sqrt{175}$ | (p) $\frac{4}{3}\sqrt{243}$ |

2. Simplify:

For example $5\sqrt{3} + \sqrt{20} - 2\sqrt{12} + \sqrt{45} = 5\sqrt{3} + 2\sqrt{5} - 4\sqrt{3} + 3\sqrt{5} = (5 - 4)\sqrt{3} + (2 + 3)\sqrt{5} = \sqrt{3} + 5\sqrt{5}$

- | | | | |
|--|--|--|--|
| (a) $\sqrt{3} + 2\sqrt{3} + 4\sqrt{3}$ | (b) $3\sqrt{5} + 5\sqrt{5} - 2\sqrt{5}$ | (c) $5\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$ | (d) $\sqrt{18} + \sqrt{32} - \sqrt{2}$ |
| (e) $\sqrt{75} + \sqrt{32} - \sqrt{27}$ | (f) $\sqrt{50} + \sqrt{63} - \sqrt{28}$ | (g) $4\sqrt{48} - 5\sqrt{27}$ | (h) $3\sqrt{24} - 5\sqrt{54}$ |
| (i) $\sqrt{80} + 2\sqrt{245}$ | (j) $\sqrt{128} + 3\sqrt{18} - \sqrt{162}$ | (k) $4\sqrt{45} - 3\sqrt{54} - 6\sqrt{5}$ | (l) $3\sqrt{8} + 5\sqrt{18} - 9\sqrt{2}$ |
| (m) $7\sqrt{24} - 4\sqrt{5} + 2\sqrt{6}$ | (n) $3\sqrt{32} + 2\sqrt{50} - 8\sqrt{18}$ | (o) $\sqrt{128} + 4\sqrt{75} - \sqrt{162}$ | (p) $2\sqrt{363} + 5\sqrt{243} + \sqrt{192}$ |
| (q) $\sqrt{a} - 4\sqrt{a} + 2\sqrt{a}$ | (r) $3\sqrt{x} + 2\sqrt{y} - 8\sqrt{x}$ | (s) $-3\sqrt{a} + 4\sqrt{a} - 6\sqrt{b}$ | (t) $-3\sqrt{x} - 4\sqrt{y} + 4\sqrt{x}$ |

3. Express each of the following in simplest surd form.

For example $4\sqrt{2} \times \sqrt{24} = 4\sqrt{2} \times 2\sqrt{6} = 2\sqrt{12} = 16\sqrt{3}$

OR $4\sqrt{2} \times \sqrt{24} = 4\sqrt{48} = 4 \times 4\sqrt{3} = 16\sqrt{3}$

- | | | | |
|-------------------------------------|--|--|--|
| (a) $2\sqrt{3} \times 5\sqrt{2}$ | (b) $5\sqrt{2} \times 3\sqrt{6}$ | (c) $4\sqrt{5} \times 2\sqrt{3}$ | (d) $5\sqrt{5} \times \sqrt{10}$ |
| (e) $\sqrt{3} \times 2\sqrt{6}$ | (f) $2\sqrt{15} \times 4\sqrt{10}$ | (g) $2\sqrt{3} \times 2\sqrt{3}$ | (h) $(2\sqrt{5})^2$ |
| (i) $2\sqrt{5} \times 3\sqrt{7}$ | (j) $4\sqrt{3} \times \sqrt{18}$ | (k) $2\sqrt{8} \times \sqrt{12}$ | (l) $4\sqrt{5} \times \sqrt{20}$ |
| (m) $(\sqrt{5} + 2)(2\sqrt{5} + 3)$ | (n) $(2\sqrt{2} - \sqrt{6})(2\sqrt{3} - 1)$ | (o) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ | (p) $(2\sqrt{3} - 5)(2\sqrt{3} + 3)$ |
| (q) $(\sqrt{7} + 1)(2\sqrt{7} - 5)$ | (r) $(\sqrt{3} + 1)^2$ | (s) $(2\sqrt{6} + \sqrt{3})^2$ | (t) $(2\sqrt{6} - \sqrt{3})(2\sqrt{6} + \sqrt{3})$ |
| (u) $(\sqrt{x} - \sqrt{y})^2$ | (v) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ | (w) $(2\sqrt{a} + \sqrt{b})(\sqrt{a} - 5\sqrt{b})$ | (x) $\sqrt{x}(2\sqrt{y} + \sqrt{x})$ |

4. Find the values of the pronumerals.

- | | | |
|--|---|--|
| (a) $6 - \sqrt{2} = a - \sqrt{b}$ | (b) $x + \sqrt{y} = 3 + \sqrt{10}$ | (c) $2a + 3\sqrt{b} = 4 + 3\sqrt{2}$ |
| (d) $a\sqrt{3} = \sqrt{27}$ | (e) $\sqrt{21} + 3 = \sqrt{3}(\sqrt{a} + \sqrt{b})$ | (f) $(\sqrt{2} + \sqrt{5})^2 = a + b\sqrt{10}$ |
| (g) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = c$ | (h) $x + y\sqrt{3} = (2\sqrt{3} - 1)(\sqrt{3} + 2)$ | (i) $\sqrt{125} - \sqrt{x} = 2\sqrt{5}$ |

5. Express each of the following in simplest surd form with rational denominator.

For example $\frac{6}{5\sqrt{3}} = \frac{6}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{15} = \frac{2\sqrt{3}}{5}$

- | | | | |
|---------------------------|----------------------------------|---------------------------------|----------------------------------|
| (a) $\frac{1}{\sqrt{2}}$ | (b) $\frac{3}{\sqrt{5}}$ | (c) $\frac{2}{\sqrt{3}}$ | (d) $\frac{1}{2\sqrt{3}}$ |
| (e) $\frac{1}{5\sqrt{2}}$ | (f) $\frac{2}{3\sqrt{2}}$ | (g) $\frac{2}{\sqrt{5}}$ | (h) $\frac{3}{\sqrt{2}}$ |
| (f) $\frac{5}{2\sqrt{3}}$ | (g) $\frac{4\sqrt{3}}{\sqrt{2}}$ | (h) $\frac{\sqrt{a}}{\sqrt{b}}$ | (i) $\frac{\sqrt{x}}{\sqrt{2y}}$ |

6. Multiply each surd by its conjugate and then simplify:

- | | | | |
|-----------------------------|-----------------------------|-----------------------|------------------------------|
| (a) $(\sqrt{5} + \sqrt{2})$ | (b) $(\sqrt{3} + \sqrt{2})$ | (c) $(\sqrt{2} + 1)$ | (d) $(\sqrt{8} - \sqrt{2})$ |
| (e) $(\sqrt{6} + 2)$ | (f) $(3\sqrt{2} + 1)$ | (g) $(2\sqrt{5} - 1)$ | (h) $(5\sqrt{2} + \sqrt{3})$ |

7. Express the following surds with a rational denominator.

For example $\frac{\sqrt{3}}{\sqrt{5} + 1} = \frac{\sqrt{3}}{\sqrt{5} + 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} - 1} = \frac{\sqrt{3}(\sqrt{5} - 1)}{(\sqrt{5})^2 - 1^2} = \frac{\sqrt{15} - \sqrt{3}}{4}$

- | | | | |
|---|---|---|---|
| (a) $\frac{1}{\sqrt{2} + 1}$ | (b) $\frac{1}{\sqrt{3} - 1}$ | (c) $\frac{\sqrt{2}}{\sqrt{5} + \sqrt{2}}$ | (d) $\frac{2}{\sqrt{3} + 1}$ |
| (e) $\frac{4}{\sqrt{6} - \sqrt{2}}$ | (f) $\frac{\sqrt{5}}{\sqrt{5} - 2}$ | (g) $\frac{\sqrt{3}}{2\sqrt{3} + 1}$ | (h) $\frac{2\sqrt{6}}{\sqrt{6} - 2}$ |
| (i) $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} + 1}$ | (j) $\frac{\sqrt{3} + 1}{\sqrt{5} - 2\sqrt{2}}$ | (k) $\frac{1}{\sqrt{3} + \sqrt{2}}$ | (l) $\frac{6}{7 - 2\sqrt{5}}$ |
| (m) $\frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} + \sqrt{2}}$ | (n) $\frac{3\sqrt{2} - 1}{3\sqrt{2} + 1}$ | (o) $\frac{\sqrt{5} + \sqrt{3}}{2\sqrt{10} - \sqrt{6}}$ | (p) $\frac{2\sqrt{2} - 5\sqrt{3}}{3\sqrt{6} - \sqrt{15}}$ |