MATH141 - Worksheet
LOGARITHMS

Definition: Let b and N be positive real numbers. The **logarithm to base b of N** is given by x where

$$x = \log_b N \iff N = b^x$$

Note that a logarithm is simply an *index* or *exponent*.

$$\log_b xy = \log_b x + \log_b y \qquad \log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^p = p \log_b x \qquad \log_a a = 1 \qquad log_a 1 = 0$$

Change of Base Rule: $\log_b x = \dfrac{\log_c x}{\log_c b}$

1. **Without** using your calculator, evaluate each of the following.
   (a) $\log_3 9$ (b) $\log_a a$ (c) $\log_2 128$ (d) $\log_2 32$
   (e) $\log_3 27$ (f) $\log_4 0.25$ (g) $\log_{10} 0.0001$ (h) $\log_a a^3$
   (i) $\log_5 625$ (j) $\log_3 81$ (k) $\log_5 125$ (l) $\log_2 16$
   (m) $\log_3 \dfrac{1}{27}$ (n) $\log_3 \sqrt{3}$ (o) $\log_{\sqrt{3}} 243$ (p) $\log_9 3$
   (q) $\log_8 \dfrac{1}{4}$ (r) $\log_9 243$ (s) $\log_4 \dfrac{1}{128}$ (t) $\log_4 0.125$

2. Find x in each of the following cases.
   (a) $\log_3 x = 3$ (b) $\log_x 81 = 2$ (c) $\log_5 625 = x$ (d) $\log_x 27 = 3$
   (e) $\log_9 x = 0.5$ (f) $\log_9 x = 0.25$ (g) $\log_3 27\sqrt{3} = x$ (h) $\log_7 x = 2$

3. Simplify each of the following expressions.
   (a) $\log_2 16 + \log_2 8$ (b) $\log_{10} 2 + \log_{10} 5$ (c) $(\log_2 16)(\log_2 4)$ (d) $\log_3 54 - \log_3 18$
   (e) $\dfrac{\log_a 8}{\log_a 2}$ (f) $\log_a 5 + \log_a \frac{1}{5}$ (g) $\log_{10} 125 + \log_{10} 32 - \log_{10} 4$
   (h) $\log_2 18 - 2\log_2 3$ (i) $\frac{1}{2}\log_{10} 16 + 2\log_{10} 5$ (j) $\log_2(2^x)$ (k) $\dfrac{\log_{10} 25}{\log_{10} 5}$

4. Without using a calculator, solve each of the following equations for x.
   (a) $\log_{10} x = \log_{10} 4 + \log_{10} 2$ (b) $\log_{10} x = \log_{10} 4 - \log_{10} 2$ (c) $\log_{10} x = \frac{1}{2}\log_{10}\left(\frac{1}{4}\right)$
   (d) $3\log_{10} 2 + \log_{10} 3 = \log_{10} x$ (e) $\log_2 16 + \log_2 x = \log_2 32$
   (f) $\log_{10} x^2 = 2$ (g) $2\log_{10} x + 3 = \log_{10}(x^5)$ (h) $\log_{10} 2 + \log_{10} 5 + \log_{10} x - \log_{10} 3 = 2$
   (i) $2\log_{10} x + 3 = 5\log_{10} x$ (j) $\log_{10} x = 4\log_{10} 2 - 2\log_{10} x$ (k) $\log_{10} x - \log_{10}(x-1) = 1$

5. Evaluate each of the following, to four decimal places, using the change of base formula and a calculator.
   (a) $\log_2 5$ (b) $\log_3 12$ (c) $\log_5 20$ (d) $\log_4 3$
   (e) $\log_3 16$ (f) $\log_6 4$ (g) $\log_5 3$ (h) $\log_3 5$

6. If $x = \log_{10} 2$ and $y = \log_{10} 3$, express the following in terms of x and y.
   (a) $\log_{10}\left(\frac{2}{3}\right)$ (b) $\log_{10} 6$ (c) $\log_{10} 4$ (d) $\log_{10} 9$
   (e) $\log_{10}\left(\frac{4}{9}\right)$ (f) $\log_{10} 60$ (g) $\log_{10} 36$ (h) $\log_{10} 54$