

Family Name	_____
First Name	_____
Student Number	_____
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University of Wollongong
School of Mathematics and Applied Statistics
MATH 141 — MATHEMATICS 1C, PART 1
Autumn Session Examination 2004

Time Allowed: 3 hours and 15 minutes

Number of Questions: 4.

Directions to Candidates

1. Each question is to be attempted.
2. The four questions are of equal value (individual parts within a question may not be of equal value).
3. The examination paper is printed on both sides.
4. Four solution books are provided. The solution to each question is to be submitted in its own separate, clearly labelled, solution book.
5. WORKING (including all necessary reasoning) is to be shown for all solutions.
6. All notation is as used in lectures. A Table of Integrals is attached.

Examination Materials/Aids Allowed

Non-alphanumeric, non-programmable, calculators are permitted.

A one-page, double-sided, A4 size summary sheet is permitted.

Examination Materials/Aids to be supplied

None.

This examination paper must NOT be removed from the examination room.

Question 1 (Use a separate book for your answers to Question 1.)

(a) Consider the following matrices.

$$A = (a_{ij}) = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \qquad B = (b_{ij}) = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$
$$C = (c_{ij}) = \begin{pmatrix} -2 & 3 & -1 \\ 4 & -6 & 2 \end{pmatrix} \qquad D = (d_{ij}) = \begin{pmatrix} 3 & 5 \\ 1 & 4 \end{pmatrix}$$

(i) Evaluate $\sum_{n=1}^2 \sum_{k=1}^3 c_{n2} a_{k1} \delta_{nk}$.

(ii) If possible, evaluate $(A - D)C$ and $C^T B$.

Where the calculation is not possible, give a brief reason.

(iii) Let $\underset{\sim}{x} = \begin{pmatrix} x \\ y \end{pmatrix}$. If possible, find the solution to the homogeneous system $C^T \underset{\sim}{x} = \underset{\sim}{0}$.

(iv) Find the rank of the matrix B .

(v) Calculate the determinant of D , $|D|$, and determine whether D^{-1} exists or not. Do **NOT** find D^{-1} .

(vi) Show that the eigenvalues of A are $\lambda = 1$ and $\lambda = 5$. Find the eigenvectors corresponding to the eigenvalue $\lambda = 1$.

(b) Consider the system of equations $A\underset{\sim}{x} = \underset{\sim}{b}$, where

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 2 \\ 1 & -2 & -1 \end{pmatrix}, \quad \underset{\sim}{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{and} \quad \underset{\sim}{b} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}.$$

(i) Reduce the augmented matrix $(A|I|\underset{\sim}{b})$ to echelon form, $(A^E|I^E|\underset{\sim}{b}^E)$.

(ii) Write down the elementary matrices, E_1 , E_2 , E_3 , corresponding to the first three row operations performed in part (b)(i).

(iii) Reduce the augmented matrix $(A|I|\underset{\sim}{b})$ to reduced echelon form, $(A^R|I^R|\underset{\sim}{b}^R)$.

(iv) If possible, find A^{-1} and the solution to the system, $\underset{\sim}{x}$.

(v) If $\underset{\sim}{x}$ was found in part (b)(iv), verify the value for y using Cramer's Rule.

Question 2 (Use a separate book for your answers to Question 2)

(a) Simplify $\frac{\sqrt{10}}{\sqrt{110}}$.

(b) Solve for all possible values of x if $\cos 3x = -\frac{1}{2}$.

(c) Find the *exact* value of $\cos y$ if $y = \operatorname{arccot} 3$ and $0 \leq y \leq \frac{\pi}{2}$.

(d) If $f(w) = 2w^2$ and $g(y) = \frac{1}{\sqrt{1+y^2}}$ find $f(g(x))$ and $g(f(3))$.

(e) Find the domain of the function $f(x) = \frac{2x-3}{x^2-4}$.

(f) Evaluate the following limits, if they exist.

(i) $\lim_{x \rightarrow 0} \cos x$.

(ii) $\lim_{x \rightarrow 3} \frac{x^4 - 9}{x^2 - 3}$.

(iii) $\lim_{x \rightarrow -\infty} \frac{2004x^3 - 2001}{2004x^5 + 2005}$.

(iv) $\lim_{x \rightarrow \infty} \frac{19x^3 + 18x}{21x^3 - 35}$.

(g) (i) Is $g(x) = \frac{1}{4x}$ the inverse of the function $f(x) = 4x$? You **must** justify your answer.

(ii) Does the function $f(x) = \frac{3x}{x-2}$ have an inverse? You **must** justify your answer.

If it does, find the domain and range of the inverse function $f^{-1}(x)$.

Question 3 (*Use a separate book for your answers to Question 3.*)

(a) Let $\underline{a} = (1, -1, 3)$, $\underline{b} = (4, -2, 1)$ and $\underline{c} = (2, 1, -6)$.

(i) Find the unit vector of \underline{a} and $\sqrt{11}\underline{a}$.

(ii) Determine if the vectors \underline{b} and \underline{c} are perpendicular.

(iii) Determine the direction angles, α, β and γ of \underline{c} .

(iv) Find the component of \underline{a} on \underline{b} and that of \underline{a} on $-\underline{b}$.

(v) Find the projection of \underline{a} on \underline{b} and that of \underline{a} on $-\underline{b}$.

(b) (i) Find the equation of the line, \mathcal{L}_1 , passing through the points $(2, 1, -1)$ and $(0, -1, 2)$.

(ii) Find the equation of the line, \mathcal{L}_2 , passing through the point $(1, -1, 2)$ and parallel to $\underline{a} = (-1, 1, 2)$.

(iii) Determine whether the lines \mathcal{L}_1 and \mathcal{L}_2 , found in parts (b)(i) and (b)(ii) respectively, intersect, are parallel or are skew.

If the lines intersect, find the point of intersection. If the lines are skew, find the distance between them.

(iv) The equation of the line \mathcal{L}_3 is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} t.$$

Find the point of intersection between the lines \mathcal{L}_1 , found in part (b)(i), and \mathcal{L}_3 .

(c) (i) Find an equation, in vector parametric form, representing the plane \mathcal{P}_1 passing through the points $(1, -1, 3)$, $(2, -1, -2)$ and $(2, 3, -3)$.

(ii) Show that the linear form of the equation representing \mathcal{P}_1 , found in part (c)(i), is given by

$$20x + y + 4z = 31.$$

(iii) Let the linear form of the equation representing the plane \mathcal{P}_2 be given by

$$2x - 3y + z - 3 = 0.$$

Determine whether the planes \mathcal{P}_1 , found in part (c)(i), and \mathcal{P}_2 intersect or are parallel. If the two planes intersect, find the equation of the intersection line. If the two planes are parallel, find the distance between them.

(iv) Is the point $(1, 1, 1)$ on the plane \mathcal{P}_2 ? Give reasons for your answer.

Question 4 (*Use a separate book for your answers to Question 4.*)

(a) Differentiate the following functions with respect to x , showing all your working.

(i) $x + x^3$

(ii) $\sin^2 x$

(iii) $\frac{\cosh x}{2 + \sinh x}$

(iv) $-\cos(x^2)$

(b) If y is given implicitly by the equation $y(x + 1) - y^2 = \tan x$ find $\frac{dy}{dx}$ in terms of x and y .

(c) A curve is defined by the parametric equation $x(t) = t^2$ and $y(t) = e^t$.

Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(d) (i) Plot the following points, given in polar form

$$A\left(3, \frac{\pi}{3}\right), \quad B\left(-3, \frac{4\pi}{3}\right), \quad C\left(3, \frac{-\pi}{3}\right).$$

(ii) Express the polar equation $r = \frac{6}{3 \cos \theta + 2 \sin \theta}$ in Cartesian coordinates.

(e) Evaluate the following integrals.

(i) $\int (4 + 7x - \cos x) dx$

(ii) $\int_1^{e^2} \frac{1}{x} dx$

(iii) $\int e^{18x} \sin(4x) dx$

(iv) $\int (7 + e^x) \sqrt{e^x + 7x} dx$

(v) $\int \frac{\cosh x}{5 + \sinh x} dx$