MATH141 - Worksheet
ALGEBRAIC FRACTIONS

To simplify algebraic fractions:

1. Factorise numerator and denominator.
2. Cancel any common factors.

1. Simplify the following fractions.

For example $\frac{m^2+3m-10}{m^2-4m+4} = \frac{(m+5)(m-2)}{(m-2)(m-2)} = \frac{m+5}{m-2}$

(a) $\frac{2xy}{5yz}$ (b) $\frac{a^2b}{ab^2}$ (c) $\frac{x-5}{7x-35}$ (d) $\frac{x^2-1}{x+1}$

(e) $\frac{a^3-1}{a-1}$ (f) $\frac{a^2+b^2+2ab}{a+b}$ (g) $\frac{4-2a}{a^2-2a}$ (h) $\frac{m^2+2m-8}{m^2-4}$

(i) $\frac{3x^2y-6xy}{2x^2y-4xy^2}$ (j) $\frac{ab}{a^2b^2-ab}$ (k) $\frac{a^2-5a}{a^2-4a-5}$ (l) $\frac{6a^2-8ab}{9ab-12b^2}$

(m) $\frac{3x^2-12}{10-5x}$ (n) $\frac{x^2+3x+2}{x^2+5x+6}$ (o) $\frac{6x^2-150}{3x+15}$ (p) $\frac{8a^2-8}{4a^2+8a+4}$

(q) $\frac{a^3-27}{a^2-9}$ (r) $\frac{x^2-x-6}{x^2-10x+21}$ (s) $\frac{2x^2-7x-15}{x^2-25}$ (t) $\frac{(a+b)^2-c^2}{3a+3b-3c}$

(u) $\frac{x^2-5x+6}{x^2+x-12}$ (v) $\frac{x^2+4x+4}{x^2-3x-10}$ (w) $\frac{4x^3y-16xy}{x^2+2x-8}$ (x) $\frac{m^2+m-2}{m^2-m}$

To **multiply** fractions:

1. Cancel any common factors.
2. Multiply numerators together and then denominators.

To **divide** fractions:

1. Multiply by the *reciprocal*.
2. Proceed as for multipliying.

2. Simplify

For example $\frac{x^2-5x+6}{x^2-9} \times \frac{x^2+3x}{x^2-x-2} = \frac{(x-2)(x-3)}{(x-3)(x+3)} \times \frac{x(x+3)}{(x-2)(x+1)} = \frac{x}{x+1}$

(a) $\frac{4a}{7} \times \frac{21}{5a}$ (b) $\frac{2m}{3n} \times \frac{6}{m^2}$ (c) $\frac{16x}{5} \times \frac{25y}{8x}$ (d) $\frac{18}{7x} \div \frac{9}{28x}$

(e) $\frac{3ab}{4a} \times \frac{8a^2}{12b}$ (f) $\frac{1}{x+1} \times \frac{3x+3}{4}$ (g) $\frac{10x-15}{6} \times \frac{1}{8x-12}$ (h) $\frac{4}{2-9x} \div \frac{8}{14-6x}$

(i) $\frac{3m-6}{4} \times \frac{8m}{m^2-2m}$ (j) $\frac{4a-6}{5} \div \frac{6a-9}{15}$ (k) $\frac{m^2-mn}{n^2-n} \times \frac{n-1}{m-n}$ (l) $\frac{1-a}{b+b^2} \div \frac{1-a^2}{1-b^2}$

(m) $\frac{a-b}{8} \times \frac{16}{b^2-a^2}$ (n) $\frac{y^2-y}{3y^3+3y^2} \div \frac{y^2-1}{6y}$ (o) $\frac{x^2-4}{2x-4} \times \frac{2}{x+2}$ (p) $\frac{a^2+5a+6}{a^2-25} \div \frac{a+3}{a-5}$

(q) $\frac{5x+15}{x+1} \div \frac{x+3}{2x^2+x-1}$ (r) $\frac{x-3y}{x^3y} \div \frac{3y-x}{xy^3}$ (s) $\frac{m^3+m^2}{x^2-x} \times \frac{x-x^3}{m+1}$ (t) $\frac{x^2-y^2}{x^2-2xy+y^2} \times \frac{xy-y^2}{xy+y^2}$

(u) $\frac{a^2-2a}{b^2-4b+4} \div \frac{a}{b-2}$ (v) $\frac{x^2-7x+10}{x^2-10x+25} \div \frac{x-5}{x-2}$ (w) $\frac{x^2-x-20}{x^2-25} \div \frac{x+1}{x^2+5x}$ (x) $\frac{6x^3+6}{x-2} \times \frac{x^2-4}{3x^2-3x+3}$

(y) $\frac{2a^2-3ab}{ab-b^2} \times \frac{2a^2-2ab}{4a-6b}$ (z) $\frac{15x^2-5xy}{10xy} \div \frac{3x-y}{2y}$

Addition and subtraction of algebraic fractions:

1. Find the lowest common multiple of the denominator.
2. Write each fraction using the common denominator.
3. Add (or subtract) the numerators.

3. Express each of the following as a single fraction.

Examples $\frac{3x}{5} - \frac{2x-1}{7} = \frac{21x}{35} - \frac{5(2x-1)}{35} = \frac{21x-10x+5}{35} = \frac{11x+5}{35}$

$\frac{3}{5x} + \frac{2}{7x} = \frac{21}{35x} + \frac{10}{35x} = \frac{21+10}{35x} = \frac{31}{35x}$

(a) $\frac{x}{5} - \frac{x}{6}$ (b) $\frac{3x}{8} + \frac{x}{2}$ (c) $\frac{a}{3} + \frac{4a}{5} - \frac{a}{6}$ (d) $\frac{y}{2} + \frac{2y}{3} - \frac{y}{4}$

(e) $\frac{a+2}{5} - \frac{a-1}{3}$ (f) $\frac{2x-y}{3} - \frac{x-3y}{6}$ (g) $\frac{3x+2}{6} - \frac{x+1}{4}$ (h) $\frac{3m-2n}{5} + \frac{m}{1}$

(i) $\frac{x}{2} + \frac{y}{4} - \frac{x+y}{3}$ (j) $\frac{a-2b}{6} - \frac{2a+b}{9}$ (k) $\frac{3(a+b)}{4} - \frac{a-b}{6}$ (l) $\frac{5}{x} + \frac{4}{x}$

(m) $\frac{3}{a} + \frac{b}{a}$ (n) $\frac{1}{x} - \frac{2}{3x}$ (o) $\frac{3}{2a} + \frac{5}{3a}$ (p) $\frac{5}{7y} + \frac{2}{y}$

(q) $\frac{3}{a} + \frac{1}{a^2}$ (r) $\frac{1}{ab} - \frac{2}{b}$ (s) $\frac{m}{n} - \frac{n}{m}$ (t) $\frac{4}{xy} + \frac{3}{yz}$

(u) $\frac{5}{a^2b} - \frac{2}{ab^2}$ (v) $\frac{1}{ab} + \frac{a}{bc}$ (w) $\frac{a+1}{6a} + \frac{a-4}{2a}$ (x) $\frac{1}{x} + \frac{2}{x} - \frac{1}{x^2}$

4. Express each of the following as a single fraction.

Examples $\frac{3}{x^2-4} + \frac{1}{x-2} = \frac{3}{(x-2)(x+2)} + \frac{x+2}{(x-2)(x+2)} = \frac{3+x+2}{(x-2)(x+2)} = \frac{x+5}{(x-2)(x+2)}$

$\frac{1}{x-y} - \frac{1}{x+y} = \frac{x+y}{(x-y)(x+y)} - \frac{x-y}{(x-y)(x+y)} = \frac{x+y-(x-y)}{(x-y)(x+y)} = \frac{2y}{(x-y)(x+y)}$

(a) $\frac{1}{x} + \frac{2}{x+y}$ (b) $\frac{2}{a+1} + \frac{1}{a-3}$ (c) $\frac{1}{x+2} - \frac{1}{x+3}$ (d) $\frac{3}{a+4} + \frac{2}{a-1}$

(e) $\frac{a}{b-c} - \frac{3a}{5b-5c}$ (f) $\frac{5c}{a^2+ab} - \frac{c}{a+b}$ (g) $\frac{5}{(x+1)^2} - \frac{2}{x+1}$ (h) $\frac{y}{x-y} - \frac{y^2}{x^2-y}$

(i) $\frac{3}{b^2-4} - \frac{5}{3b-6}$ (j) $\frac{3}{x-y} - \frac{2}{x+y}$ (k) $\frac{x}{x-y} + \frac{y}{x+y}$ (l) $\frac{1}{x^2-4} - \frac{1}{x+2}$

(m) $\frac{3}{(x-2)^2} + \frac{2}{x-2}$ (n) $\frac{3}{x-2} + \frac{1}{x+3}$ (o) $\frac{5}{2a+6} + \frac{a}{a^2-9}$ (p) $\frac{6}{3x-2} - \frac{8}{4x+}$

(q) $\frac{y}{x^2-xy} + \frac{1}{x}$ (r) $\frac{1}{x+2} + \frac{1}{x-2} + \frac{4}{x^2-4}$ (s) $\frac{3}{x^2-4} - \frac{2}{x^2-3x+2}$ (t) $\frac{3a+1}{3a-1} - \frac{3a-}{3a+}$

NOTE:

$$\frac{a}{b/c} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$$

$$\frac{a/b}{c} = \frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$$

$$\frac{a+m}{b+m} \neq \frac{a}{b}$$