2006 Q2

#1

a) Walks 35.2 metres in one minutes

Walks (35.2×60) metres in one hour. $\boxed{1}$

Walks $\dfrac{(35.2\times 60)}{1000}$ kilometres in one hour. (1½)

Average speed is $\underline{\underline{2.112}}$ kilometres per hour. (1½) $\boxed{1}$

only 1.5 for 2.1 or 2 without stating d.p

1 mark for 21.12

b) $\dfrac{1-\dfrac{y^2}{x^2}}{1+\dfrac{y^2}{x^2}} = \dfrac{\dfrac{x^2-y^2}{x^2}}{\dfrac{x^2+y^2}{x^2}} = \dfrac{x^2-y^2}{x^2+y^2}$ $\boxed{2}$

c(i) In first quadrant

$\sin(x) = \dfrac{1}{2} \iff \underline{x = \pi/6}$ $\boxed{1}$

S	A
T	C

So, other angle is in the second quadrant. Equivalent angle is $\pi - w$

$\pi - \dfrac{\pi}{6} = \underline{\underline{\dfrac{5\pi}{6}}}$ $\boxed{1}$

-½ mark for each wrong answer if 2 correct answers.

c(ii) $y = \sin^{-1}(1/2)$

$\sin y = \frac{1}{2}$ with $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

So $\underline{y = \pi/6}$ $\boxed{1}$

only ½ if $\frac{\pi}{6}$ & $\frac{5\pi}{6}$.

(c iii) $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$ [1/2 mark]

$\sin^2\left(\frac{\pi}{8}\right) = \sin^2\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{1-\cos\pi/4}{2}$ [$\frac{1}{2}$] mark

$= \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$

Note $\frac{\pi}{8}$ is in the 1st quadrant So $\sin\left(\frac{\pi}{8}\right) > 0$

Thus $\sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)} \equiv \frac{1}{2}\sqrt{2-\sqrt{2}}$ [1]

(d) $h(x) = x^3 - 7x + 6$

$h(1) = 1 - 7 + 6 = 0 \Rightarrow (x-1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
\hline
x-1 & x^3 + 0x^2 - 7x + 6 \\
 & x^3 - x^2 \\
\hline
 & \quad x^2 - 7x \\
 & \quad x^2 - x \\
\hline
 & \qquad -6x + 6 \\
 & \qquad -6x + 6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$h(x) = (x-1)(x^2+x-6)$

$= (x-1)(x+3)(x-2)$

So zeroes are $x=-3$, $x=1$ and $x=2$ [3 marks]

$2\frac{1}{2}$ for factorising correctly but not stating the zeroes

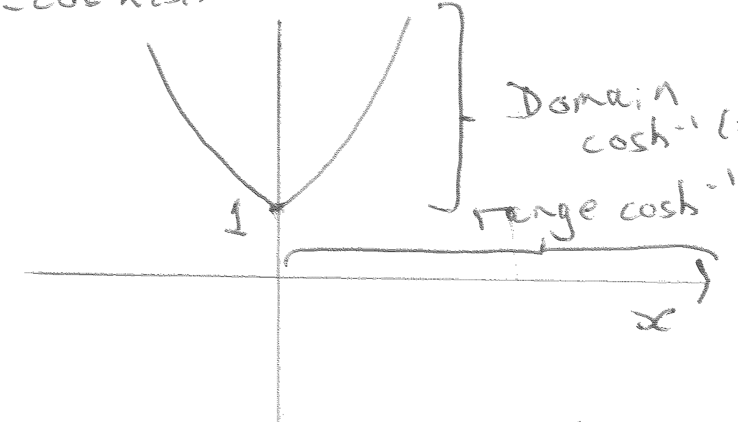
(e i)

$$\tanh^{-1} x = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$$ ½ mark

$$\tanh^{-1} 0.5 = \frac{1}{2} \ln \left| \frac{3/2}{1/2} \right| = \underline{\frac{1}{2} \ln 3}$$ ½ mark

only ½ if don't leave exact answer.

(ii) $y = \cosh(x)$



1 mark

only ½ if don't mark the '1' on graph

In order to construct the inverse we need to restrict the domain of $\cosh x$

dom $\cosh(x) = x \in [0, \infty)$

range $\cosh(x) = y \in [1, \infty)$

dom $\cosh^{-1}(x) = \underline{x \in [1, \infty)}$ ½ mark

range $\cosh^{-1}(x) = \underline{y \in [0, \infty)}$ ½ mark

f (i) $\lim_{x \to c} \frac{f(x)}{3g(x)} = \frac{2}{3(5)} = \frac{2}{15}$ (1 mark)

(ii) $f(x) = 4x$, $g(x) = x$

1 mark 1 mark

(1 mark if satisfy ~~two~~ one of the conditions)

½.

f(iii) Q2 #4

$$\lim_{x \to \infty} \frac{4x^2 + x}{x^2 + 2x + 1} = \lim_{x \to \infty} \frac{\frac{1}{x^2}(4x^2 + x)}{\frac{1}{x^2}(x^2 + 2x + 1)}$$ 1 mark for dividing by x^2

$$= \lim_{x \to \infty} \frac{(4 + 1/x)}{(1 + 2/x + 1/x^2)}$$

$$= \underline{\underline{4}}$$ 1 mark

g(i) $h(x) = \underline{\underline{\sqrt{\sqrt{x} - 1}}}$ 1 mark

$\underline{\underline{\text{Range } h(x) = [0, \infty)}}$ 1 mark

$\underline{\underline{\text{Dom } h(x) = x \geq 1}}$ * 1 mark

(ii) Let $z = x^2$

$y = \sinh^{-1} z$ $\qquad z = x^2$

$\frac{dy}{dz} = \frac{1}{\sqrt{1 + z^2}}$ 1/2 mark $\qquad \frac{dz}{dx} = 2x$ 1/2 mark

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \underline{\underline{\frac{2x}{\sqrt{1 + x^4}}}}$$ 1 mark

* Note Dom $(f \circ g)(x) = \left\{ x : x \in \text{Dom } g \text{ and } g(x) \in \text{Dom } f \right\}$

Dom $g(x) = x \geq 1$
Range $g(x) = y \geq 0$
Dom $f(x) = x \geq 0$