Algebra 2007

(i)

1) $A^T = \begin{pmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & 1 & 7 \end{pmatrix}$ 1

1

ii) $\begin{vmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & 1 & 7 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \\ 0 & 2 & 4 \end{vmatrix}$

$= - \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0$ 2

iii) $\left(\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 1 & -1 & 1 & 0 \\ 2 & 3 & 7 & -5 \end{array}\right)$

$R_1 \leftrightarrow R_2$ $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 2 & 3 & 7 & -5 \end{array}\right)$

$R_3 \to R_3 - 2R_1$ $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 5 & 5 & -5 \end{array}\right)$

3

$R_3 \to R_3 - 5R_2$ $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

2 iv) $z = t$, $y = -1 - t$

$x = y - z = -1 - 2t$ 2

$$\underline{x} = \begin{pmatrix} -1-2t \\ -1-t \\ t \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

v) $\underline{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ 1

vi) $E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ $E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{pmatrix}$ 2

vii) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & 1 & 7 \end{pmatrix}$ 1

b) i) $\begin{vmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{vmatrix}$

$$= (1-\lambda)(-2-\lambda) - 4 = 0$$

$$\lambda^2 + 2\lambda - \lambda - 2 - 4 = 0$$ 1

$$\lambda^2 + \lambda - 6 = 0$$

ii) $(\lambda + 3)(\lambda - 2) = 0$ 1

$\lambda = 2, -3$

iii) $\lambda = 2$ $\left(\begin{array}{cc|c} -1 & 2 & 0 \\ 2 & -4 & 0 \end{array}\right)$ $y = t$, $x = 2t$ $(2, 1)^T$ 2

$\lambda = -3$

$$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \qquad \begin{matrix} y = 2t \\ x = -t \end{matrix} \qquad (-1, 2)^T$$

iv)

$$B^2 = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 0 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -2 \\ -2 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} - 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

2

v)

$$B + I - 6B^{-1} = 0$$

$$B^{-1} = \frac{1}{6}(B + I)$$

2

vi) $$B^{-1} = \begin{pmatrix} 2/6 & 2/6 \\ 2/6 & -1/6 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -1/6 \end{pmatrix}$$

2

vii) $$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -1/6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

1

viii) i) $B\underline{x} = \underline{0} \qquad B^{-1}B\underline{x} = 0 \Rightarrow \underline{x} = 0$

ii) $$\underline{x} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -1/6 \end{pmatrix}\begin{pmatrix} 3 \\ -6 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

2

Question 3 (*Use a separate book for your answers to Question 3. Failing to use a separate answer book may mean that your answer is not marked.*)

(a) Let $\vec{a} = (-1, 2, 2)$, $\vec{b} = (-3, 0, 4)$ and $\vec{c} = (4, 1, 3)$.

(i) Find the unit vector of $\vec{a}$ and $3\vec{a}$. [1½ + 1½ = 3]

(ii) Determine if vectors $\vec{b}$ and $\vec{c}$ are perpendicular. [2]

(iii) Find the projection of $\vec{a}$ on $\vec{b}$ and that of $\vec{a}$ on $-2\vec{b}$. [1½ + 1½ = 3]

[8]

(b) (i) Find the vector parametric equation of the line $\mathcal{L}_1$ passing through the points $P(3, -1, 0)$ and $Q(1, -2, 1)$. [2]

(ii) Given the line $\mathcal{L}$ defined by

$$\mathcal{L}: \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} s,$$

[1 + 2 + 1 = 4]

determine if the lines $\mathcal{L}$ and $\mathcal{L}_1$ intersect, are parallel or are skew, where $\mathcal{L}_1$ is the line found in (b)(i).

(iii) For the line $\mathcal{L}$ given in part (b)(ii), find the distance between $\mathcal{L}$ and the point $R(2, 1, 1)$. [3]

[

(c) (i) Consider the three points $A(1, 1, 1)$, $B(2, -1, -2)$ and $C(3, -1, 2)$. Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AB} \times \overrightarrow{AC}$. [½ + ½ + 2 = 3]

(ii) Using (c)(i), represent the plane $\mathcal{P}_1$ that passes through the points $A(1, 1, 1)$, $B(2, -1, -2)$ and $C(3, -1, 2)$ in vector parametric form. [2]

(iii) Using (c)(i), show that the linear form of the equation representing $\mathcal{P}_1$, found in part (c)(ii), is given by

$$8x + 7y - 2z = 13.$$

[3]

NOTE: (a)(i) { Correct $\hat{a}$ + wrong $\widehat{3a}$ = [2]
$|\vec{a}| + |3\vec{a}|$ = [1]

(iii) { Correct formula = [1]
Correct $\vec{a}$ on $\vec{b}$ + wrong $\vec{a}$ on $2\vec{b}$ = [2]

(b) (i) { Correct answer = [2]
Correct approach but numerical error = [1]

(ii) { Not Parallel = [1]
Correct System + Correct Conclusion = [2]
Final Conclusion = [1]

(c)(i) { Correct $\overrightarrow{AB}$ + $\overrightarrow{AC}$ = [1]
Correct $\overrightarrow{AB} \times \overrightarrow{AC}$ = [2], Numerical error lost = [1]

(c)(ii) {Correct equation = [2], "even if got $\vec{AB}$ and $\vec{AC}$ may have numerical error"
(iii) {Identifying $\vec{AB} \times \vec{AC}$ as normal = [2]
writing the Cartesian equation = [1]
correct d = [1] }

Question 4 (*Use a separate book for your answers to Question 4. Failing to use a separate answer book may mean that your answer is not marked.*)

(a) (i) Differentiate the function $y = \frac{1}{5}\left(7x^3 - \frac{1}{2}x^2 + 2x - 3\right)$ with respect to x

(ii) Differentiate the function $y = x \tan x$ with respect to x.

(iii) By writing $\operatorname{sech} x$ as $(\cosh x)^{-1}$ show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

(iv) Differentiate the function $y = \cos(\sin x)$ with respect to x.

(v) Differentiate the function $y = x^{\sin x}$.

(b) ~~Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ if~~ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x + \sin x}{2y - \sin y}$.

(c) A curve is defined by the parametric equation $x(t) = t\cos(t)$ and $y(t) = \sin(t)$. Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

(d) (i) Find the value(s) of r when $\theta = \frac{6.5\pi}{12}$ if $r = 5\cos 3\theta$.

(ii) Using the polar graph paper provided at the end of the exam paper sketch the function $r = 5\cos 3\theta$, $0 \le \theta \le 2\pi$. The following set of values may be useful

θ	$\frac{0\pi}{12}$	$\frac{1\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$	$\frac{7\pi}{12}$	$\frac{8\pi}{12}$	$\frac{9\pi}{12}$	$\frac{10\pi}{12}$	$\frac{11\pi}{12}$	$\frac{12\pi}{12}$	$\frac{13\pi}{12}$
r	5	3.5	0	-3.5	-5	-3.5	0	3.5	5	3.5	0	-3.5	-5	-3.5
θ	$\frac{14\pi}{12}$	$\frac{15\pi}{12}$	$\frac{16\pi}{12}$	$\frac{17\pi}{12}$	$\frac{18\pi}{12}$	$\frac{19\pi}{12}$	$\frac{20\pi}{12}$	$\frac{21\pi}{12}$	$\frac{22\pi}{12}$	$\frac{23\pi}{12}$	$\frac{24\pi}{12}$			
r	0	3.5	5	3.5	0	-3.5	-5	-3.5	0	3.5	5			

You should also use your answer to (d)(i).

(e) Evaluate the following integrals.

(i) $\displaystyle\int_2^3 3e^{2x}\,\mathrm{d}x$

(ii) $\displaystyle\int e^{18x}\sin 4x\,\mathrm{d}x$

(iii) $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{19x+6}}$

(iv) $\displaystyle\int_2^4 (2x-3)^5\,\mathrm{d}x$

(v) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_{75}^{\tan x} t^2 \sin t\,\mathrm{d}t$

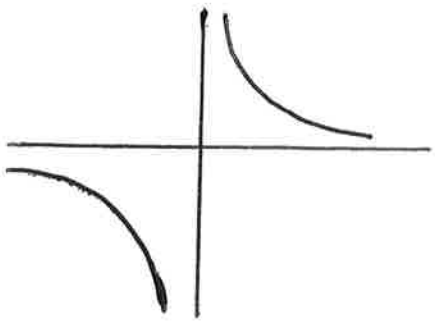
2007

Q2.

a) $\sqrt{27} \cdot \sqrt{3} = \sqrt{81} = \underline{\underline{9}}$ 1 mark

b) $2x^2 + x - 1 = \underline{\underline{(2x-1)(x+1)}}$ 1 mark

c)



1 mark

d) $6x^3y^{-2} \times \frac{1}{24} x^{-5} y^4 = \frac{6}{24} \frac{x^3}{x^5} \frac{y^4}{y^2}$

$= \frac{1}{4} \cdot \frac{1}{x^2} \cdot y^2$

$= \underline{\underline{\left(\frac{y}{2x}\right)^2}}$ 2 marks

e) $\frac{10! \times 15}{(60 \times 60 \times 24)} = 630 \text{ days}$ 2 marks

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2 marks

f)

$$
\begin{array}{r|l}
 & 2x^2 - 2x - 1 \\
\hline
x - 0.5 & 2x^3 - 3x^2 + 0.x + 0.5 \\
 & 2x^3 - x^2 \\
\hline
 & -2x^2 + 0.x \\
 & -2x^2 + x \\
\hline
 & -x + 0.5 \\
 & -x + 0.5 \\
\hline
 & 0
\end{array}
$$

$$2x^2 - 2x - 1 = 0$$

$$\Rightarrow \quad x = \frac{2 \pm \sqrt{4 + 8}}{4}$$

$$= \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$$

$$= \frac{1 \pm \sqrt{3}}{2}$$

2 marks

Zeroes are $x = \dfrac{1 \pm \sqrt{3}}{2}$ and $x = 0.5$ 4 marks

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gi) $\lim\limits_{x \to 0} (x-2)(x-3) = -2(-3) = \underline{\underline{6}}$ 1 mark

ii) $f(x) = x$, $g(x) = x^2$. 3 marks

(there is more than one answer)

iii) $\lim_{x \to -\infty} \dfrac{2x^2+3x+4}{5x^2+4x+3} = \lim_{x \to -\infty} \dfrac{\frac{1}{x^2}(2x^2+3x+4)}{\frac{1}{x^2}(5x^2+4x+3)}$

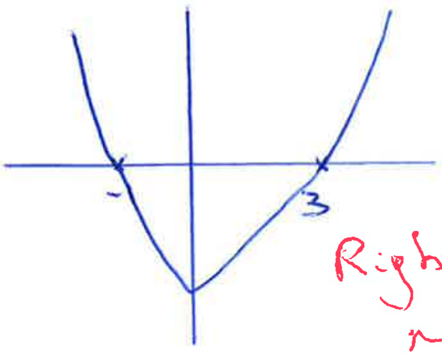
$= \lim_{x \to -\infty} \left(\dfrac{2 + 3/x + 4/x^2}{5 + 4/x + 3/x^2} \right) = \underline{\underline{\dfrac{2}{5}}}$ 2 marks

h i) $\cosh^2 x - \sinh^2 x = 1$ 1 mark

$\sinh x = 4 \Rightarrow \cosh^2 x = 17$ 1 mark

$\underline{\underline{\cosh x = \sqrt{17}}}$

i i) $f(x) = x^2 - 2x - 3 = (x-3)(x+1)$



This function does <u>NOT</u> have an inverse as it is not 1-1 2 marks

Right answer, no reason = 0.5 mark.

iii) Let $z = x^2 + x$ $\quad y = \cos^{-1} z$

$\dfrac{dz}{dx} = 2x+1$ 1 mark $\quad \dfrac{dy}{dz} = \dfrac{-1}{\sqrt{1-z^2}}$ 1 mark

$\dfrac{dy}{dx} = \dfrac{dz}{dx} \cdot \dfrac{dy}{dz} = \underline{\underline{\dfrac{-1}{\sqrt{1-(x^2+x)^2}} \cdot (2x+1)}}$ 1 mark

1 mark 4 marks

Question 4

(a) Let $\vec{a} = (-1, 2, 2)$, $\vec{b} = (-3, 0, 4)$ and $\vec{c} = (4, 1, 3)$.

(i) Find the unit vector of $\vec{a}$ and $3\vec{a}$.

(ii) Determine if vectors $\vec{b}$ and $\vec{c}$ are perpendicular.

(iii) Find the projection of $\vec{a}$ on $\vec{b}$ and that of $\vec{a}$ on $-2\vec{b}$.

(b) (i) Find the vector parametric equation of the line $\mathcal{L}_1$ passing through the points $P(3, -1, 0)$ and $Q(1, -2, 1)$.

(ii) Given the line $\mathcal{L}$ defined by

$$\mathcal{L}: \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} s,$$

determine if the lines $\mathcal{L}$ and $\mathcal{L}_1$ intersect, are parallel or are skew, where $\mathcal{L}_1$ is the line found in (b)(i).

(iii) For the line $\mathcal{L}$ given in part (b)(ii), find the distance between $\mathcal{L}$ and the point $R(2, 1, 1)$.

(c) (i) Consider the three points $A(1, 1, 1)$, $B(2, -1, -2)$ and $C(3, -1, 2)$. Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Using (c)(i), represent the plane $\mathcal{P}_1$ that passes through the points $A(1, 1, 1)$, $B(2, -1, -2)$ and $C(3, -1, 2)$ in vector parametric form.

(iii) Using (c)(i), show that the linear form of the equation representing $\mathcal{P}_1$, found in part (c)(ii), is given by

$$8x + 7y - 2z = 13.$$

Questions and Solutions

Solutions of Vector Geometry Questions

M141 Final Exam - 2007

Q.3

(a) $\vec{a} = (-1, 2, 2)$, $\vec{b} = (-3, 0, 4)$, $\vec{c} = (4, 1, 3)$

(i) The unit vector of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{\vec{a}}{\sqrt{(-1)^2 + 2^2 + 2^2}} = \frac{\vec{a}}{\sqrt{9}}$$

$$= \frac{\vec{a}}{3} = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right).$$

The unit vector of $3\vec{a} = \dfrac{3\vec{a}}{|3\vec{a}|} = \dfrac{3\vec{a}}{3|\vec{a}|}$

$$= \frac{\vec{a}}{|\vec{a}|} = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right).$$

The unit unit vectors of $\vec{a}$ and $3\vec{a}$ are same.

(ii) The vector $\vec{b}$ is perpendicular with the $\vec{c}$ if $\vec{b} \cdot \vec{c} = 0$.

Observe that $\vec{b} \cdot \vec{c} = -3 \times 4 + 0 \times 1 + 4 \times 3$

$$= -12 + 12 = 0$$

$\therefore \vec{b} \perp \vec{c}$.

3(a)(ii) The projection of $\vec{a}$ on $\vec{b}$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

$$= \frac{-1\times-3+2\times0+2\times4}{\left(\sqrt{(-3)^2+0^2+4^2}\right)^2}\vec{b}$$

$$= \frac{3+8}{25}\vec{b} = \frac{11}{25}\vec{b}$$

$$= \left(-\frac{33}{25}, 0, \frac{44}{25}\right).$$

The projection of $\vec{a}$ on $-2\vec{b}$ is

$$= \frac{\vec{a}\cdot(-2\vec{b})}{|-2\vec{b}|^2}(-2)\vec{b}$$

$$= \frac{-2(\vec{a}\cdot\vec{b})}{4|\vec{b}|^2}\times(-2)\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

$$= \left(-\frac{33}{25}, 0, \frac{44}{25}\right).$$

3(b)(i) Given $P(3,-1,0)$ and $Q(1,-2,1)$.

$\therefore \quad \overrightarrow{PQ} = (1-3, -2-(-1), 1-0) = (-2,-1,1)$

Therefore the vector parametric equation of the line $\mathcal{L}_1$ passing through the given points is

$$\mathcal{L}_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}.$$

(ii) The vector parametric equation of the line $\mathcal{L}_2$ that is passing through the origin and parallel to the vector $\vec{a} = (-4,3,2)$ is given by

$$\mathcal{L}_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix}, \quad s \in \mathbb{R}.$$

(iii) Given $\mathcal{L} : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$.

~~Sol~~ The vector $\vec{b} = (-1,2,1)$ and $\overrightarrow{PQ}$ (obtained in (i)) are not parallel as

$\frac{-2}{-1} \neq \frac{-1}{2}$.

Therefore the lines L_1 and L may either intersect or skew lines.

Suppose ~~that~~ the lines L and L_1 do intersect. Then there is a common point on both L and L_1.

Suppose $R(x_0, y_0, z_0)$ is a point on L and L_1. Therefore,

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$ (as $R(x_0, y_0, z_0)$ lie on L)

and

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$ ($\because$ R lies on L_1)

Thus we have,

$$\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -s \\ 2+2s \\ 1+s \end{pmatrix} = \begin{pmatrix} 3-2t \\ -1-t \\ t \end{pmatrix}, \quad s, t \in \mathbb{R}.$$

$$\left.\begin{aligned} \Rightarrow \quad -s &= 3-2t \quad \cdots (i) \\ 2+2s &= -1-t \quad \cdots (ii) \\ 1+s &= t \quad \cdots (iii) \end{aligned}\right\}$$

From equation (i), we obtain

$$s = 2t-3$$

Substituting $s = 2t-3$ in (ii) yields,

$$2+2(2t-3) = -1-t$$

$$\Rightarrow 2+4t-6 = -1-t$$

$$\Rightarrow -4+4t = -1-t$$

$$\Rightarrow 5t = 3 \Rightarrow t = \frac{3}{5}.$$

$$\therefore s = 2\times\frac{3}{5}-3 = \frac{6}{5}-3 = \frac{6-15}{5} = \frac{9}{5}$$

Substituting $t = \frac{3}{5}$ and $s = \frac{9}{5}$ in (iii),

we obtain

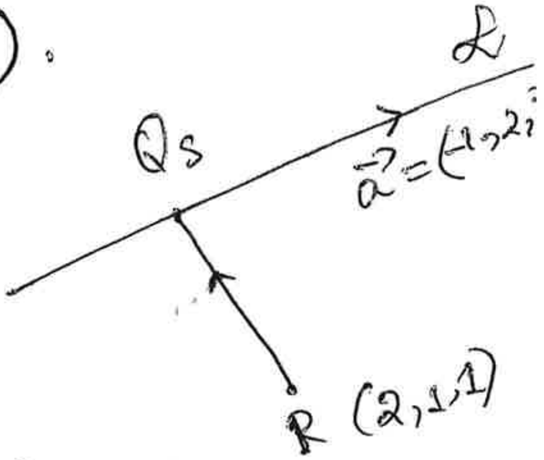
$1+\frac{9}{5} = \frac{3}{5}$, which is impossible.

$\therefore$ The lines L and L_1 do not intersect. Thus the lines L and L_1 are skew.

3 (b) (iv) $\mathcal{L}: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}. \quad s \in \mathbb{R}$

Therefore, a point on $\mathcal{L}$ is given by

$Q_s = (-s, 2+2s, 1+s).$



$\therefore \overrightarrow{RQ_s} = (s-2, 1+2s, s)$

$\overrightarrow{RQ_s} \cdot \vec{a} = 0$

$\Rightarrow (s-2, 1+2s, s) \cdot (-1, 2, 1) = 0$

$\Rightarrow -(s-2) + 2(1+2s) + s = 0$

$\Rightarrow -s + 2 + 2 + 4s + s = 0$

$\Rightarrow 4 + 4s = 0$

$\Rightarrow s = -1$

$\therefore$ For $s = -1$, $\overrightarrow{RQ_s} \perp \vec{a}$

$\therefore$ The distance of R from $\mathcal{L}$

$= |\overrightarrow{RQ_s}|$ for $s = -1$

$= |(-1-2, 1-2, -1)| = \sqrt{(-3)^2 + (-1)^2 + (-1)^2}$

$= \sqrt{11}$

3 (c)(i) $A(1,1,1)$, $B(2,-1,-2)$, $C(3,-1,2)$

$\therefore \overrightarrow{AB} = (2-1, -1-1, -2-1) = (1,-2,-3)$

$\overrightarrow{AC} = (3-1, -1-1, 2-1) = (2,-2,1)$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -3 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= (-2-6)\vec{i} - (1+6)\vec{j} + (-2+4)$$

$$= -8\vec{i} - 7\vec{j} + 2\vec{k} = (-8,-7,2)$$

(ii) The required equation of the plane is given by (in parametric form)

$$\begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = \lambda \overrightarrow{AB} + \mu \overrightarrow{AC}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}$$

(iii) The equation of the line in linear form is given by

$(x-1, y-1, z-1) \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = 0$

$$-8(x-1) - 7(y-1) + 2(z-1) = 0$$

$$\Rightarrow -8x + 8 - 7y + 7 + 2z - 2 = 0$$

$$\Rightarrow -8x + \ldots$$

$$\Rightarrow 8x + 7y - 2z = 13.$$

a) i) $y = \frac{1}{5}\left(7x^3 - \frac{1}{2}x^2 + 2x - 3\right)$

$$\frac{dy}{dx} = \frac{1}{5}\left(21x^2 - x + 2\right)$$

1 mark

ii) $y = x\tan x$

Use product rule

$u = x \qquad v = \tan x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \sec^2 x$

$$\frac{dy}{dx} = \tan x + x\sec^2 x$$

2 marks

iii) $y = \frac{1}{\cosh x}$ Use Quotient Rule

$u = 1 \qquad v = \cosh x$

$\frac{du}{dx} = 0 \qquad \frac{dv}{dx} = \sinh x$

$$\frac{dy}{dx} = \frac{\cosh x \cdot 0 - 1 \cdot \sinh x}{\cosh^2 x} = \frac{-\sinh x}{\cosh x} \cdot \frac{1}{\cosh x}$$

$$= -\tanh x \cdot \operatorname{sech} x$$

2 marks

iv) Use Chain Rule

$\left.\begin{aligned} u &= \sin x \\ \frac{du}{dx} &= \cos x \end{aligned}\right\} \frac{1}{2} \qquad \left.\begin{aligned} y &= \cos u \\ \frac{dy}{du} &= -\sin u \end{aligned}\right\} \frac{1}{2}$ mark

2 marks

$$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du} = -\cos x \cdot \sin(\sin x)$$

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a i)

$$y = x^{\sin x}$$

$$= e^{\ln x^{\sin x}} = e^{\sin x \ln x}$$ 1 mark

Let $z = \sin x \ln x$

$$\frac{dz}{dx} = \cos x \ln x + \frac{\sin x}{x}$$ (using the product rule)

$$\frac{dy}{dx} = \left[\cos x \ln x + \frac{\sin x}{x}\right] e^{\sin x \ln x}$$

$$= \underline{\underline{\left[\cos x \ln x + \frac{\sin x}{x}\right] x^{\sin x}}}$$ 2 marks

b)

$$\frac{dy}{dx} = \frac{x + \sin x}{2y - \sin y}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{x+\sin x}{2y-\sin y}\right)$$ 1 mark

1 mark $$= \frac{(2y - \sin y)(1 + \cos x) - (x + \sin x)(2 - \cos y)\frac{dy}{dx}}{(2y - \sin y)^2}$$ [

$$= \frac{(2y - \sin y)(1 + \cos x) - \frac{(x + \sin x)^2 (2 - \cos y)}{2y - \sin y}}{(2y - \sin y)^2}$$

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b) $$\frac{d^2y}{dx^2} = \frac{(2y-\sin y)^2(1+\cos x) - (x+\sin x)^2(2-\cos y)}{(2y-\sin y)^3}$$

2 marks

c) $$\left.\begin{aligned} x &= t\cos t \\ \frac{dx}{dt} &= \cos t - t\sin t \end{aligned}\right\} \tfrac{1}{2} \qquad \left.\begin{aligned} y &= \sin t \\ \frac{dy}{dt} &= \cos t \end{aligned}\right\} \tfrac{1}{2}$$

$$\left.\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}\right\} \tfrac{1}{2} = \frac{\cos t}{\cos t - t\sin t} = \frac{1}{1 - t.\tan t}$$ 2 marks

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{1-t.\tan t}\right) = \left.\frac{d}{dt}\left(\frac{1}{1-t.\tan t}\right)\cdot\frac{dt}{dx}\right]$$ 1 mark

$$= \frac{[1-t.\tan t].0 + 1\left[\tan t + t\sec^2 t\right]}{(1-t.\tan t)^2} \cdot \frac{1}{(\cos t - t\sin t)}$$

$$= \frac{\tan t + t\sec^2 t}{(1-t.\tan t)^2} \cdot \frac{1}{\cos t - t\sin t}$$ 2 marks

d i) $$r = 5\cos(3\theta)$$

$$= 5\cos\left(\frac{6.5\pi}{4}\right)$$ ½ mark

$$= \underline{1.91}$$ 1 mark

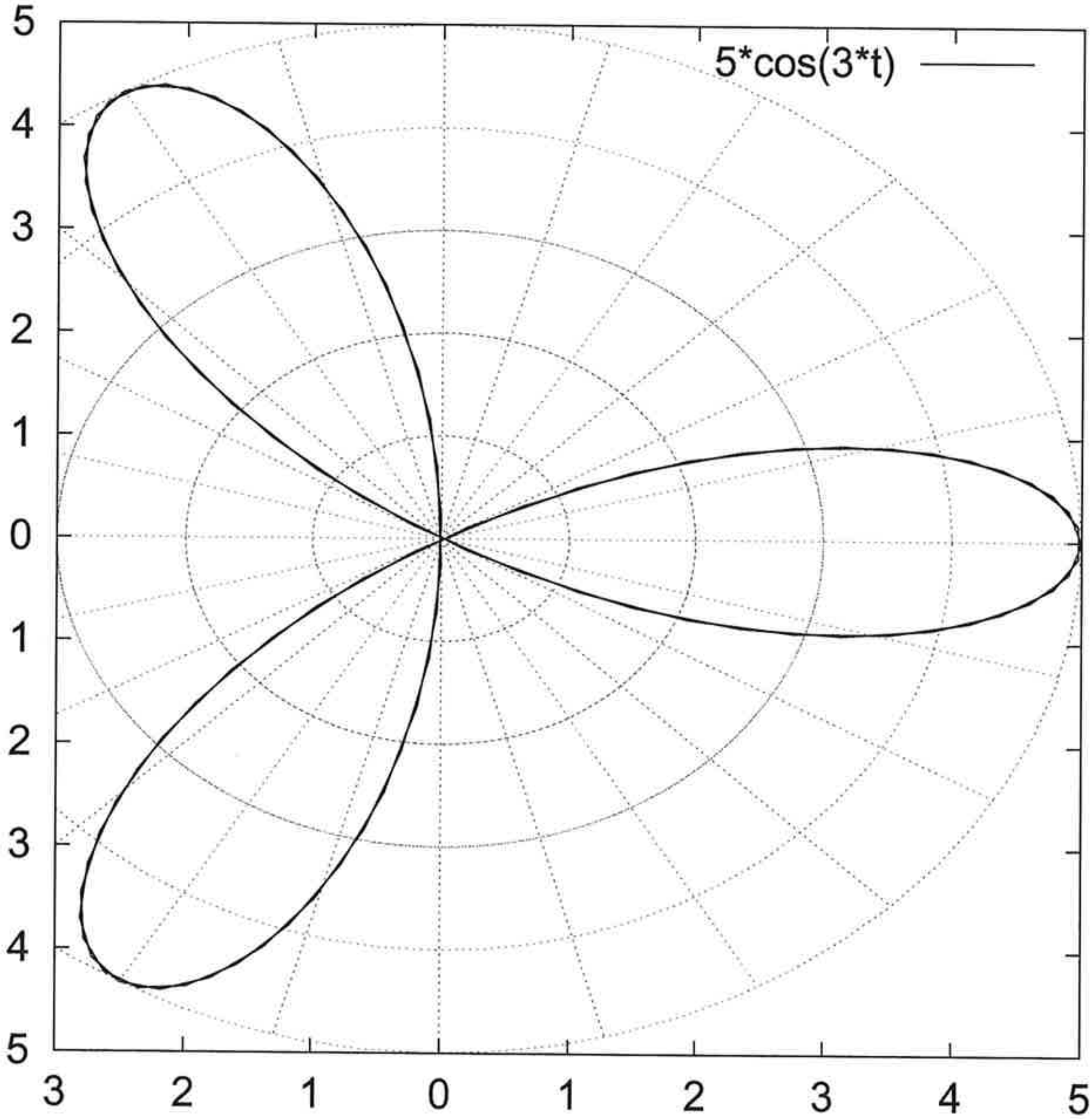

Figure 1: Plot of $r = 5\cos 3\theta$.

iv) $I = \int_2^4 (2x-3)^5 \, dx$ 2 marks

Let $v = 2x-3$ $x=2 \Rightarrow v=1$
$dv = 2dx$ $x=4 \Rightarrow v=5$ $\}\ 1/2$

$\Rightarrow \frac{dv}{2} = dx \}\ 1/2$

1/2 1/2

$I = \int_1^5 \frac{v^5}{2} \, dv = \left[\frac{v^6}{12}\right]_1^5 = \frac{(5^6-1)}{12} = \underline{\underline{1302}}$

v) $I = \frac{d}{dx} \int_a^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x)$ – 1 mark

$f(t) = t^2 \sin t$ $g(x) = \tan(x)$ – 1/2 mark
$g'(x) = \sec^2 x$

$I = \underline{\underline{\tan^2 x \sin[\tan(x)] \cdot \sec^2 x}}$ 2 marks