

Answers to 2004 Vectors Question

Question 3 (a)

(i)  $\hat{a} = \begin{pmatrix} \frac{1}{\sqrt{11}} \\ \frac{-1}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \end{pmatrix}$  and  $\sqrt{11}a = \begin{pmatrix} \sqrt{11} \\ -\sqrt{11} \\ 3\sqrt{11} \end{pmatrix}$ .

(ii) Yes,  $\underline{b}$  and  $\underline{c}$  are perpendicular.

(iii)  $\alpha = \cos^{-1} \frac{2}{\sqrt{41}}$ ,  $\beta = \cos^{-1} \frac{1}{\sqrt{41}}$ ,  $\gamma = \cos^{-1} \frac{-6}{\sqrt{41}}$ .

(iv) Both answers are  $\frac{9}{\sqrt{21}}$  (using definition from Adam's lectures).

(v) Both answers are  $\begin{pmatrix} \frac{12}{7} \\ \frac{-6}{7} \\ \frac{3}{7} \end{pmatrix}$ .

(b)

(i)  $\mathcal{L}_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} t, \quad t \in \mathbb{R}.$

(ii)  $\mathcal{L}_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} s, \quad s \in \mathbb{R}.$

(iii) Lines are skew. Distance between them is  $\frac{7}{\sqrt{66}}$  units.

(iv)  $\begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}$ .

(c)

(i)  $\mathcal{P}_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} t + \begin{pmatrix} 1 \\ 4 \\ -6 \end{pmatrix} s, \quad t, s \in \mathbb{R}.$

(ii) Just show the three points known to be on  $\mathcal{P}_1$  all satisfy the linear form equation given.

(iii) The planes are not parallel, so they meet in a line. The equation of the line is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{48}{31} \\ \frac{1}{31} \\ 0 \end{pmatrix} + \begin{pmatrix} -13 \\ 12 \\ 62 \end{pmatrix} u, \quad u \in \mathbb{R}.$$

(iv) No, because its coordinates do not satisfy the linear form equation for  $\mathcal{P}_2$ .